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Chiral and Gluon Condensates at Finite Temperature

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Abstract
We investigate the thermal behaviour of gluon and chiral condensates within an effective Lagrangian of pseudoscalar mesons coupled to a scalar glueball. This Lagrangian mimics the scale and chiral symmetries of QCD.

1. INTRODUCTION

The construction of effective models for low energy QCD is strongly constrained by global symmetry aspects. The most important one is the chiral symmetry which, together with its spontaneous breakdown at low energies, was therefore already extensively studied. Another symmetry of the classical QCD–Lagrangian in the limit of vanishing quark current masses is the dilatation symmetry or scale invariance. This symmetry exhibits an anomaly, i.e. it is broken on the quantum level by radiative corrections [1].

An effective realisation of the QCD scale anomaly was found in the early 80's [2], by adding to the classical Lagrangian a scalar glueball field $\mathcal{X}$ with an interaction potential of the form

$$V(\mathcal{X}) = C\mathcal{X}^4 \left[\ln\left(\mathcal{X}/\mathcal{X}_p\right) - 1/4\right] \ . \tag{1}$$

Here C and $\mathcal{X}_p$ are parameters to be specified later. This leads to $\theta^\mu_\mu = -C\mathcal{X}^4$ for the trace of the energy–momentum tensor which is also the divergence of the dilatation current [3]. This quantity with scaling dimension 4 can then be identified with the trace anomaly in QCD,

$$\langle \theta^{\mu\,QCD}_\mu \rangle = \langle \frac{\beta(g)}{2g} G^a_{\mu\nu} G^{a\,\mu\nu} \rangle \equiv -\langle C\mathcal{X}^4 \rangle \ . \tag{2}$$

A revival of this idea came with the articles of Campbell et. al. [4] who, somewhat controversially, suggested to regard $\langle\mathcal{X}\rangle$ as an order parameter for a deconfining phase transition. Similar to the restoration of spontaneous broken symmetries at high temperature and/or densities, deconfinement should signal itself in the effective potential (EP) of the $\mathcal{X}$–field by a phase transition from $\langle\mathcal{X}\rangle = \mathcal{X}_0 \neq 0$ to $\langle\mathcal{X}\rangle_{T_c} = 0$.

The breaking of the two symmetries in low–energy QCD mentioned above is connected with the appearance of two important vacuum condensates, the chiral condensate $\langle q\bar{q}\rangle$ and the gluon condensate $\langle G^a_{\mu\nu} G^{a\,\mu\nu}\rangle$, respectively. Therefore these expectation values have been considered as order parameters for symmetry restoration.

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Campbell et al. [4] conjectured a strong correlation between the behaviour of these two condensates at rising temperature or density. The main relation, under the assumption of factorisation, is [4]

$$\langle q\bar{q}\rangle \propto \left(\frac{\langle \mathcal{X}\rangle}{\mathcal{X}_0}\right)^3 \langle \mathrm{Tr}(U+U^\dagger)\rangle \,, \tag{3}$$

with U denoting the chiral matrix field, defined below. This means that a vanishing of $\langle \mathcal{X}\rangle$, i.e. of the gluon condensate, drives also the chiral condensate to zero. Therefore the important question is whether the temperature scale for vanishing of the chiral condensate is dominated by the intrinsic chiral dynamics, which drives the disappearance of $\langle \mathrm{Tr}(U+U^\dagger)\rangle$, or by the vanishing of $\langle \mathcal{X}\rangle$.

The idea in [4] has triggered a lot of (sometimes rather phenomenological) work investigating the scaling behaviour of low energy Lagrangians at finite temperature or density [6]–[12]. Here, we will follow the original suggestion in [4] to determine the thermal behaviour of $\langle \mathcal{X}\rangle$ via the minimum of the effective potential, by performing a systematic calculation of the latter. We will see that the conjectures in [4] about the character of the chiral and deconfinement phase transitions are only partially supported by our detailed calculation.

We start from the nonlinear σ–model, extended by a scalar glueball field in order to mimic the scaling properties of QCD [4]. For this effective low–energy Lagrangian we calculate in Sec. 2 the Gaussian effective potential (GEP) for the dilaton field $\mathcal{X}$. From the GEP we can read off the temperature dependence of the expectation value $\langle \mathcal{X}\rangle$, leading via the identification (2) to the thermal behaviour of the gluon condensate. This will be done in Sec. 3, where we also discuss the influence of thermal excitations of the $\mathcal{X}$–field on the chiral condensate. In Sec. 4 we will discuss our results.

2. THE GAUSSIAN EFFECTIVE POTENTIAL

We start with the same Lagrangian as in [4], but neglecting the contribution of the η' meson:

$$\mathcal{L}^{eff} = \frac{f_\pi^2}{4}\left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^2 \mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + \frac{c f_\pi^2}{2}\left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^3 \mathrm{Tr}\left[\mathcal{M}_q(U+U^\dagger)\right] + \frac{1}{2}\partial_\mu \mathcal{X}\partial^\mu \mathcal{X} - V(\mathcal{X}) . \tag{4}$$

Here

$$U = \exp\left(\mathrm{i}\sum_{a=1}^{N^2-1} \lambda^a \,\frac{\Phi^a}{f_\pi}\right) \quad , \quad \mathcal{M}_q = \mathrm{diag}\ (m_u,\ m_d,\ m_s,\ \ldots) \quad , \quad \mathcal{X}_0 = \langle \mathcal{X}\rangle_{T=0}, \tag{5}$$

λ^a are the generators of flavour SU(N) (N is the number of quark flavours), and $V(\mathcal{X})$ was defined in (1).

For $\mathcal{X} = \mathrm{const.} = \mathcal{X}_0$ this Lagrangian reduces, up to an additative constant, to the non–linear σ–model [13]. As long as the explicit chiral symmetry breaking by the quark mass term $\mathcal{M}_q$ is small, terms containing higher derivatives of U can be neglected in comparison with the first term in (4), and (4) is a good approximation to the chiral dynamics of the

$N^2 - 1$ Goldstone bosons of QCD at low energy. We will thus consider the cases $N = 2$ and $N = 3$.

We want to calculate the EP $V_{\text{eff}}(\bar{\mathcal{X}})$ with $\bar{\mathcal{X}} = \langle \mathcal{X} \rangle$. We expect a breakdown of the model in the limit $\bar{\mathcal{X}} \to 0$: The expansion of the logarithmic tree level potential $V(\mathcal{X})$ (1) in a power series near $\mathcal{X} = 0$ leads to singular coefficients for powers larger than 3. At the origin we will also pick up singularities in coupling constants related to the chiral field U, because of the effective scaling of all U–terms in (4) with powers of $\mathcal{X}$ (see below). This means that the theory cannot be expanded into a Taylor series around $\bar{\mathcal{X}} = 0$. As mentioned in [4] a particle interpretation at the origin is thus not possible. Campbell et al. interpreted this phenomenon as indication that in the phase where $\bar{\mathcal{X}} = 0$ the physical degrees of freedom of the model, the Goldstone bosons Φ^a and the glueballs $\mathcal{X}$, are no longer relevant, and that this phase corresponds to a deconfined phase where these mesons have dissolved into their quark content.

The EP implements radiative corrections to the tree level potential $V(\mathcal{X})$ arising from virtual and (at non–zero temperature) real excitations of the degrees of freedom in the Lagrangian. Since the latter are no longer well-defined in a state where $\overline{\mathcal{X}} = 0$, the EP cannot be perturbatively calculated near $\overline{\mathcal{X}} = 0$. Thus a direct comparison of the effective potential (= free energy) at the origin $\bar{\mathcal{X}} = 0$ and at the local minimum of $V_{\text{eff}}(\bar{\mathcal{X}})$ is not possible in any perturbative approach. Therefore, it is for example impossible to decide whether the phase transition connected with the disappearance of the scalar condensate $\langle \mathcal{X} \rangle$ is of first or second order. This point was apparently missed in Ref. [4]. However, we can still calculate the shift of the local minimum of $V_{\text{eff}}(\bar{\mathcal{X}})$ at $\overline{\mathcal{X}} \neq 0$ as a function of temperature and investigate at which temperature scale visible thermal effects set in which could indicate the disappearance of the gluon condensate.

We also encounter another familiar problem connected with the EP in the case of broken symmetries. The standard loop expansion [14] for the EP leads to complex contributions for values of $\bar{\mathcal{X}}$, for which the tree level potential $V(\overline{\mathcal{X}})$ is concave, i.e. its second derivative is negative. This problem has triggered a large amount of work, ranging from an interpretation of the imaginary part as a sign for instability for the corresponding $\overline{\mathcal{X}}$ state [15] to techniques which avoid the complex contributions altogether [16, 17].

In [15] values of $\overline{\mathcal{X}}$ for which $V_{\text{eff}}(\overline{\mathcal{X}})$ was complex were interpreted as false vacua, and the imaginary part was related to the decay rate per unit volume of the false vacuum. One knows, however, from the definition of the effective potential that the exact EP is real [18]. Therefore we regard the appearence of complex contributions as a sign for an unsuitable expansion scheme. We are thus led to choose an approximation where all expansion coefficients are real.

We choose a variational ansatz, the Gaussian effective potential (GEP), which was extensively studied by Stevenson et al. [19]. Most other methods, which avoid the imaginary part, are only valid in distinct temperature regions. The interpolated loop expansion [17] is only tractable for low temperatures [20], while resummation techniques are valid only for high temperatures [16]. With the GEP ansatz we are able to cover the whole temperature region. A more detailed discussion of the problem of the imaginary part and a comparison of different methods to solve the problem is given in [21].

The principle of the GEP is the introduction of an arbitrary mass parameter Ω for the dilaton field $\mathcal{X}$. The effective potential is then be minimized with respect to this parameter, allowing only for real values of Ω. This is in analogy to the Ritz variational principle in quantum mechanics, where the ground state energy is the minimal energy under variation of the wave function. We will introduce the variational parameter Ω later within the path integral formalism by a method developed by Okopińska [22].

We start from the finite temperature partition function Z^β. After expanding the chiral field U, Eq. (5), in terms up to fourth order in the Goldstone fields Φ or $\partial_\mu\Phi$, we get for the partition function

$$Z^\beta[J^a,K] = \mathcal{N}\int\prod_{a=1}^{N^2-1}\mathcal{D}[\Phi^a]\,\mathcal{D}[\mathcal{X}]\exp\left\{\mathrm{i}\int_{\mathcal{C}}\mathrm{d}^4x\;\left[\mathcal{L}^{\mathrm{eff}}(\Phi^a,\partial_\mu\Phi^a,\mathcal{X},\partial_\mu\mathcal{X}) + J^a\Phi^a + K\mathcal{X}\right]\right\}. \tag{6}$$

$\mathcal{N}$ is a normalisation constant, and $\mathcal{L}^{\mathrm{eff}}$ is given as

$$\begin{aligned}
\mathcal{L}^{\mathrm{eff}} &= \left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^3 f_\pi^2 c\sum_q m_q \\
&+\frac{1}{2}\left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^2\partial_\mu\Phi^a\partial^\mu\Phi^a - \frac{1}{2}\left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^3 m_a^2\Phi^{a2} \\
&+\mathrm{Tr}\Bigg[\frac{1}{48f_\pi^2}\left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^2\sum_{abcd}^{N^2-1}\lambda^a\lambda^b\lambda^c\lambda^d\;\Big(\partial_\mu\Phi^a\,\Phi^b\,\partial^\mu\Phi^c\,\Phi^d - \partial_\mu\Phi^a\,\Phi^b\,\Phi^c\,\partial^\mu\Phi^d \\
&\qquad -\Phi^a\,\partial_\mu\Phi^b\,\partial^\mu\Phi^c\,\Phi^d + \Phi^a\,\partial_\mu\Phi^b\,\Phi^c\,\partial_\mu\Phi^d\Big)\Bigg] \\
&+\mathrm{Tr}\left[\frac{2c}{48f_\pi^2}\left(\frac{\mathcal{X}}{\mathcal{X}_0}\right)^3\sum_{abcd}^{N^2-1}\mathcal{M}_q\lambda^a\lambda^b\lambda^c\lambda^d\Phi^a\Phi^b\Phi^c\Phi^d\right] \\
&+\frac{1}{2}\partial_\mu\mathcal{X}\partial^\mu\mathcal{X} - V(\mathcal{X})\,.
\end{aligned} \tag{7}$$

J^a and K are external sources for the Goldstone bosons and dilaton fields, respectively. The first term is independent of Φ^a. Therefore we add this term to the potential $V(\mathcal{X})$ and define

$$V^\Phi(\mathcal{X}) := V(\mathcal{X}) - g_{\mathcal{X}}\mathcal{X}^3 \quad \text{with} \quad g_{\mathcal{X}} := \frac{f_\pi c}{\mathcal{X}_0^3}\sum_q m_q\,. \tag{8}$$

We will use the real–time formalism. Therefore the integration of x_0 is along the real–time path $\mathcal{C}$ in the complex plane [23]. In order to evaluate (6) we will use the sattle point approximation. This means that we expand the dilaton field $\mathcal{X}$ around the solution of the classical equation of motion

$$\partial_\mu\frac{\partial\mathcal{L}(\mathcal{X}_{cl},\Phi)}{\partial(\partial_\mu\mathcal{X}_{cl})} - \frac{\partial\mathcal{L}(\mathcal{X}_{cl},\Phi)}{\partial\mathcal{X}_{cl}} = K\,. \tag{9}$$

In (9) we substitute the field Φ by its expectation value $\langle\Phi(x)\rangle = 0$. Further we define the dilaton fluctuating field $\hat{\mathcal{X}} := \mathcal{X} - \mathcal{X}_{cl}$ and transform the measure for $\mathcal{X}$ to $\mathcal{D}[\mathcal{X}] = \mathcal{D}[\hat{\mathcal{X}}]$. Because of the starting Lagrangian (4) which contains no $\mathcal{X}-$ independent terms the kinetic term for the meson fields scales with $(\mathcal{X}_{cl}/\mathcal{X}_0)^2$:

$$\mathcal{L}_{\Phi\mathrm{kin}} = -\frac{1}{2}\Phi^a \left(\frac{\mathcal{X}_{cl}}{\mathcal{X}_0}\right)^2 \left[\Box + m_a^2 \left(\frac{\mathcal{X}_{cl}}{\mathcal{X}_0}\right)\right] \Phi^a \,, \tag{10}$$

where m_a is the meson mass in the non–linear sigma model (see eq. (26) below). In order to obtain a standard kinetic term for Φ^a we transform the meson fields Φ^a by

$$\Phi^a \longrightarrow \frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\,\Phi^a \,. \tag{11}$$

This transformation leads to an infinite Jakobi determinant for the integration measure of the Φ field,

$$\mathcal{D}[\Phi^a] \longrightarrow \mathcal{D}[\Phi^a] \times \mathrm{Det}\left[\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right)\delta^4(x-y)\right] \,, \tag{12}$$

which we absorb in the normalisation $\mathcal{N}$. After these manipulations we can write the partition function as

$$Z^\beta[J^a,K] = \mathcal{N}\int \prod_{a=1}^{N^2-1} \mathcal{D}[\Phi^a]\,\mathcal{D}[\hat{\mathcal{X}}] \times$$
$$\exp\left\{\mathrm{i}\int_{\mathcal{C}} \mathrm{d}^4x \Big[\mathcal{L}_{\mathcal{X}_{cl}} + \mathcal{L}_{\Phi\mathrm{kin}} + \mathcal{L}_{\Phi\mathrm{int}} + \mathcal{L}_{\hat{\mathcal{X}}\mathrm{kin}} + \mathcal{L}_{\hat{\mathcal{X}}\mathrm{int}} + \mathcal{L}_{\hat{\mathcal{X}}\Phi\mathrm{int}} + J^a\Phi^a + K\mathcal{X}_{cl}\Big]\right\}, \tag{13}$$

with

$$\mathcal{L}_{\mathcal{X}_{cl}} = \frac{1}{2}\partial_\mu\mathcal{X}_{cl}\partial^\mu\mathcal{X}_{cl} \,-\, V^\Phi(\mathcal{X}_{cl}) \,, \tag{14}$$

$$\mathcal{L}_{\Phi\mathrm{kin}} = -\frac{1}{2}\Phi^a \left[\Box + M_a^2(\mathcal{X}_{cl})\right] \Phi^a \,, \tag{15}$$

$$\begin{aligned}\mathcal{L}_{\Phi\mathrm{int}} = {} & \mathrm{Tr}\Bigg[\frac{1}{48 f_\pi^2}\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right)^2 \sum_{abcd}^{N^2-1} \lambda^a\lambda^b\lambda^c\lambda^d \quad \Big(\partial_\mu\Phi^a\,\Phi^b\,\partial^\mu\Phi^c\,\Phi^d - \partial_\mu\Phi^a\,\Phi^b\,\Phi^c\,\partial^\mu\Phi^d \\ & \qquad -\Phi^a\,\partial_\mu\Phi^b\,\partial^\mu\Phi^c\,\Phi^d + \Phi^a\,\partial_\mu\Phi^b\,\Phi^c\,\partial_\mu\Phi^d\Big)\Bigg] \\ & + \mathrm{Tr}\Bigg[\frac{2c}{48 f_\pi^2}\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right) \sum_{abcd}^{N^2-1} \mathcal{M}_q\lambda^a\lambda^b\lambda^c\lambda^d\Phi^a\Phi^b\Phi^c\Phi^d\Bigg] \,,\end{aligned} \tag{16}$$

$$\mathcal{L}_{\hat{\mathcal{X}}\mathrm{kin}} = -\frac{1}{2}\hat{\mathcal{X}}\Big\{\Box + \underbrace{(V^{\Phi})''(\mathcal{X}_{cl})}_{=:\,M^2_{\mathcal{X}}(\mathcal{X}_{cl})}\Big\}\hat{\mathcal{X}}\,, \tag{17}$$

$$\mathcal{L}_{\hat{\mathcal{X}}\mathrm{int}} = -\frac{1}{6}(V^{\Phi})'''(\mathcal{X}_{cl})\hat{\mathcal{X}}^3 - \frac{1}{24}V''''(\mathcal{X}_{cl})\hat{\mathcal{X}}^4 + \mathcal{O}(\hat{\mathcal{X}}^5) + \dots\,, \tag{18}$$

$$\begin{aligned}
\mathcal{L}_{\hat{\mathcal{X}}\Phi\mathrm{int}} = {} & \frac{1}{\mathcal{X}_{cl}}\hat{\mathcal{X}}\partial_\mu\Phi^a\partial^\mu\Phi^a - \frac{3}{2}\frac{1}{\mathcal{X}_{cl}}M_a^2(\mathcal{X}_{cl})\hat{\mathcal{X}}\Phi^{a\,2} \\
& + \frac{1}{2}\frac{1}{\mathcal{X}_{cl}^2}\hat{\mathcal{X}}^2\partial_\mu\Phi^a\partial^\mu\Phi^a - \frac{3}{2}\frac{M_a^2(\mathcal{X}_{cl})}{\mathcal{X}_{cl}^2}\hat{\mathcal{X}}^2\Phi^{a\,2} - \frac{1}{2}\frac{M_a^2(\mathcal{X}_{cl})}{\mathcal{X}_{cl}^3}\hat{\mathcal{X}}^3\Phi^{a\,2} \\
& + \mathrm{Tr}\Bigg[\frac{1}{48f_\pi^2}\frac{1}{\mathcal{X}_{cl}^2}\hat{\mathcal{X}}^2\sum_{abcd}^{N^2-1}\lambda^a\lambda^b\lambda^c\lambda^d \quad \Big(\partial_\mu\Phi^a\,\Phi^b\,\partial^\mu\Phi^c\,\Phi^d - \partial_\mu\Phi^a\,\Phi^b\,\Phi^c\,\partial^\mu\Phi^d \\
& \qquad - \Phi^a\,\partial_\mu\Phi^b\,\partial^\mu\Phi^c\,\Phi^d + \Phi^a\,\partial_\mu\Phi^b\,\Phi^c\,\partial_\mu\Phi^d\Big)\Bigg] \\
& + \mathrm{Tr}\Bigg[\frac{2c}{16f_\pi^2}\frac{1}{\mathcal{X}_{cl}\mathcal{X}_0}\hat{\mathcal{X}}^2\sum_{abcd}^{N^2-1}\mathcal{M}_q\lambda^a\lambda^b\lambda^c\lambda^d\Phi^a\Phi^b\Phi^c\Phi^d\Bigg] \\
& + \mathrm{Tr}\Bigg[\frac{2c}{48f_\pi^2}\frac{1}{\mathcal{X}_{cl}^2\mathcal{X}_0}\hat{\mathcal{X}}^3\sum_{abcd}^{N^2-1}\mathcal{M}_q\lambda^a\lambda^b\lambda^c\lambda^d\Phi^a\Phi^b\Phi^c\Phi^d\Bigg]\,.
\end{aligned} \tag{19}$$

At this stage of the calculation we have for the mass of the $\mathcal{X}$–field

$$M^2_{\mathcal{X}}(\mathcal{X}_{cl}) = (V^{\Phi})''(\mathcal{X}_{cl})\,. \tag{20}$$

The meson masses scale with $\sqrt{\mathcal{X}_{cl}/\mathcal{X}_0}$, and we have defined

$$M_a^2(\mathcal{X}_{cl}) := m_a^2\frac{\mathcal{X}_{cl}}{\mathcal{X}_0}\,. \tag{21}$$

Further we combine the interaction terms by

$$\mathcal{L}_{\mathrm{int}} = \mathcal{L}_{\Phi\mathrm{int}} + \mathcal{L}_{\hat{\mathcal{X}}\mathrm{int}} + \mathcal{L}_{\hat{\mathcal{X}}\Phi\mathrm{int}}\,. \tag{22}$$

We now introduce the variational parameter Ω by the method developed in [22]. This is done by adding and subtracting the quadratic term $\Omega^2\,\hat{\mathcal{X}}^2/2$ to the Lagrangian (19). We now regard $-\,\Omega^2\,\hat{\mathcal{X}}^2/2$ as a new mass term for the $\hat{\mathcal{X}}$–field and get therefore an additional contribution to the interaction part

$$\mathcal{L}_{\mathrm{int}} \longrightarrow \mathcal{L}^{\Omega}_{\mathrm{int}} = \mathcal{L}_{\mathrm{int}} - \frac{1}{2}(M^2_{\mathcal{X}} - \Omega^2)\hat{\mathcal{X}}^2\,. \tag{23}$$

This corresponds to a two point vertex, indicated by a cross in the Feynman diagrams (see Fig. 1). We thus have to evaluate the expression

$$Z^\beta[J^a, K] = \mathcal{N} \int \prod_{a=1}^{N^2-1} \mathcal{D}[\Phi^a]\, \mathcal{D}[\hat{\mathcal{X}}] \times \tag{24}$$

$$\exp\left\{ \mathrm{i} \int_{\mathcal{C}} \mathrm{d}^4x \Big[\mathcal{L}_{\mathcal{X}_{cl}} + \mathcal{L}_{\Phi\mathrm{kin}} - \frac{1}{2}\hat{\mathcal{X}}\left(\Box + \Omega^2\right)\hat{\mathcal{X}} + \mathcal{L}^{\Omega}_{\mathrm{int}} + J^a\Phi^a + K\mathcal{X}_{cl} \Big] \right\}.$$

This expression (24) is by construction independent of the arbitrary parameter Ω. But in practice, in order to evaluate (24), we have to expand the exponential containing the interaction part $\mathcal{L}^{\Omega}_{\mathrm{int}}$ and truncate the series at some point. This truncation introduces a dependence on Ω. Given such an approximation to the EP as a function of Ω, one then uses the principle of minimal sensitivity with respect to the arbitrary parameter Ω to fix $\Omega(\overline{\mathcal{X}})$ as a function of $\overline{\mathcal{X}}$ by the condition

$$\frac{\mathrm{d}V^{\mathrm{G}}_{\mathrm{eff}}(\overline{\mathcal{X}}, \Omega)}{\mathrm{d}\Omega} = 0\ . \tag{25}$$

In our present work we expand $\exp\{\mathrm{i}\int \mathrm{d}^4x \mathcal{L}^{\Omega}_{\mathrm{int}}\}$ up to first order. This means that we only include vacuum bubbles containing at most one vertex. The Feynman diagrams with one vertex which appear in our calculation are shown in Fig. 1.

We investigate three cases. Case 1 corresponds to a pure glueball scenario, neglecting in (4) all U–field terms. In case 2 we include a SU(N) flavour representation with N equal masses for the U–fields. In case 3 we consider a broken SU(3) flavour representation with $m_s \neq m_{u,d}$, denoted by SU(3)$_\mathrm{b}$. For the last case the physical meson masses in zeroth order are

$$\begin{aligned} m_\pi^2 &= \quad 2c\, m_q &&= m_1^2 = m_2^2 = m_3^2\ , \\ m_K^2 &= \quad c(m_q + m_s) &&= m_4^2 = m_5^2 = m_6^2 = m_7^2\ , \\ m_\eta^2 &= \quad 2c(m_q + 2m_s)/3 &&= m_8^2\ , \end{aligned} \tag{26}$$

where $m_q = m_u = m_d$.

The result for the partition functions for the three cases, neglecting the normalisation, is

$$\begin{aligned} Z^\beta_{\mathcal{X}}[K]\Big|_{J^a=0} = \exp\Bigg\{ \mathrm{i} \int_{\mathcal{C}} \mathrm{d}^4x\ \Bigg[& \frac{1}{2}\partial_\mu \mathcal{X}_{cl}\partial^\mu \mathcal{X}_{cl} - V(\mathcal{X}_{cl}) + K\mathcal{X}_{cl} \\ & - I_1(\Omega^2, T) - \frac{1}{2}(V''(\mathcal{X}_{cl}) - \Omega^2) I_0(\Omega^2, T) \\ & - \frac{1}{8} V''''(\mathcal{X}_{cl}) I_0(\Omega^2, T)^2 \Bigg] \Bigg\}, \end{aligned} \tag{27}$$

$$
\begin{aligned}
Z^{\beta}_{\mathrm{SU(N)}}[K]\Big|_{J^a=0} = \exp\Bigg\{ \mathrm{i}\int\limits_{\mathcal{C}} \mathrm{d}^4x \Bigg[& \frac{1}{2}\partial_\mu \mathcal{X}_{cl}\partial^\mu \mathcal{X}_{cl} - V^{\Phi}(\mathcal{X}_{cl}) + K\mathcal{X}_{cl} \\
& - I_1(\Omega^2, T) - (N^2-1) I_1(M_\pi^2(\mathcal{X}_{cl}), T) \\
& - \frac{1}{2}[(V^{\Phi})''(\mathcal{X}_{cl}) - \Omega^2] I_0(\Omega^2, T) - \frac{1}{8} V''''(\mathcal{X}_{cl}) I_0(\Omega^2, T)^2 \\
& - \frac{(N^2-1)m_\pi^2}{4Nf_\pi^2}\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right) I_0(M_\pi^2(\mathcal{X}_{cl}), T)^2 \\
& - \frac{(N^2-1)m_\pi^2}{2\mathcal{X}_{cl}\mathcal{X}_0} I_0(M_\pi^2(\mathcal{X}_{cl}), T) I_0(\Omega^2, T) \Bigg] \Bigg\} ,
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
Z^{\beta}_{\mathrm{SU(3)_b}}[K]\Big|_{J^a=0} = \exp\Bigg\{ \mathrm{i}\int\limits_{\mathcal{C}} \mathrm{d}^4x \Bigg[& \frac{1}{2}\partial_\mu \mathcal{X}_{cl}\partial^\mu \mathcal{X}_{cl} - V^{\Phi}(\mathcal{X}_{cl}) + K\mathcal{X}_{cl} \\
& - I_1(\Omega^2, T) \\
& - 3I_1(M_\pi^2(\mathcal{X}_{cl}), T) - 4I_1(M_K^2(\mathcal{X}_{cl}), T) - I_1(M_\eta^2(\mathcal{X}_{cl}), T) \\
& - \frac{1}{2}[(V^{\Phi})''(\mathcal{X}_{cl}) - \Omega^2] I_0(\Omega^2, T) - \frac{1}{8} V''''(\mathcal{X}_{cl}) I_0(\Omega^2, T)^2 \\
& - \frac{1}{24f_\pi^2}\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right) \Big[\; 9m_\pi^2 I_0(M_\pi^2(\mathcal{X}_{cl}), T)^2 \\
& \qquad - (m_\eta^2 + 4cm_s) I_0(M_\eta^2(\mathcal{X}_{cl}), T)^2 \\
& \qquad - 6m_\pi^2 I_0(M_\pi^2(\mathcal{X}_{cl}), T) I_0(M_\eta^2(\mathcal{X}_{cl}), T) \\
& \qquad + 16m_K^2 I_0(M_K^2(\mathcal{X}_{cl}), T) I_0(M_\eta^2(\mathcal{X}_{cl}), T) \Big] \\
& - \frac{1}{2\mathcal{X}_{cl}\mathcal{X}_0} I_0(\Omega^2, T) \Big[3m_\pi^2 I_0(M_\pi^2(\mathcal{X}_{cl}), T) \\
& \qquad + 4m_K^2 I_0(M_K^2(\mathcal{X}_{cl}), T) + m_\eta^2 I_0(M_\eta^2(\mathcal{X}_{cl}), T) \Big] \Bigg] \Bigg\} .
\end{aligned}
\tag{29}
$$

The integrals I_n are defined in the appendix. From the functional $Z[K] = \exp(\mathrm{i}W[K])$ we get the EP by first making a Legendre transformation

$$
\Gamma[\overline{\mathcal{X}}(x)] = W[K] - \int \mathrm{d}^4x K(x) \overline{\mathcal{X}}(x) , \tag{30}
$$

with

$$
\overline{\mathcal{X}}(x) = \frac{\delta W[K]}{\delta K(x)} . \tag{31}
$$

The expansion of the effective action $\Gamma[\overline{\mathcal{X}}(x)]$ in powers of derivatives

$$\Gamma[\overline{\mathcal{X}}(x)] = \int \mathrm{d}^4x \left[-V_{\text{eff}}(\overline{\mathcal{X}}) + \frac{1}{2}\, Z(\overline{\mathcal{X}})\, \partial_\mu \overline{\mathcal{X}}(x) \partial^\mu \overline{\mathcal{X}}(x) \, ... \right] . \tag{32}$$

contains as its first term the EP.

We apply this procedure to the partition function (27)–(29). The integrals I_n (see appendix) contain two parts, the infinite $T = 0$ contribution and the finite $T > 0$ contribution. The infinite $T = 0$ part requires a renormalisation procedure. In a strict sense, the starting Lagrangian (4) is not renormalizable because of the logarithmic interaction potential (1). But a possible regularisation of the infinite integrals would be the introduction of a cut–off parameter Λ. This regularisation is often used in effective non–renormalizable theories, where the cut–off determines the scale up to which the effective description is valid. The standard example is the Nambu and Jona–Lasinio (NJL) model.

But such a kind of additional parameter violates the scaling properties of the effective theory, which we have chosen to simulate QCD, because of the appearance of a new scale. In order to keep the wanted scaling behaviour one has to give the cut–off parameter also a conformal weight of one unit by some mechanism, as for example done for the NJL model in [8, 9].

We simplify the procedure here and cancel all infinite $T = 0$ contributions by hand. This means that we normalize the theory at tree level. We will determine the parameters C and $\mathcal{X}_p$ in (1) such that the EP at $T = 0$ reproduces the bag constant $\mathcal{B}$ and the glueball mass $m_{\mathcal{X}}$:

$$\mathcal{B} = V_{\text{eff}}(\mathcal{X} = 0) - V_{\text{eff}}(\mathcal{X} = \mathcal{X}_0) \, , \tag{33}$$

$$m_{\mathcal{X}}^2 = M_{\mathcal{X}}^2(T = 0) = V''_{\text{eff}}(\mathcal{X} = \mathcal{X}_0) \, . \tag{34}$$

As we will see later the determination of the EP at the origin $\bar{\mathcal{X}} = 0$ is not possible because of infinities in the effective coupling constant in the expansion of the logarithm. Thus a renormalisation, i.e. a determination of the parameters via (33) is not possible.

By applying (33) and (34), we fix the depth and the curvature at the minimum of the EP at $T = 0$. We are dominantly interested in the thermal behaviour and therefore in the thermal excitation around this minimum. But the characteristic of this behaviour is already mainly fixed by these two parameters (depth and curvature). Therefore we expect no drastic change in the result of the thermal properties by changing the level of normalisation in the theory.

For the value of the bag constant we choose as two limiting cases $\mathcal{B}^{1/4} = 140$ MeV and $\mathcal{B}^{1/4} = 240$ MeV which bound the common range of values appearing in the literature. We want to emphasize that the larger value is more realistic as newer results from hadron spectroscopy show [24], compared to the original work of the MIT–bag model [25] where the lower $\mathcal{B}$ was used. Also, only the larger value is compatible with the gluon condensate at $T = 0$ as extracted from QCD sum rules [26].

For the glueball mass we choose $m_{\mathcal{X}} = 1.6$ GeV, but we will also investigate other values. That value is motivated by lattice calculations [27], and also the experimental search favours candidates in the mass region of 1.5–1.8 GeV [28].

For the parameters of the chiral sector (c, m_q) we choose standard values which reproduce the physical pion mass and, for the case of $\mathrm{SU(3)_b}$, also the K and η masses in a reasonable way. We set $f_\pi = 93$ MeV.

We now apply (30)–(32) to the partition functions (27)–(29) and neglect as discussed the $T = 0$ contributions in the integrals I_n. Up to first order in the expansion of $\mathcal{L}^{\Omega}_{\text{int}}$ we are allowed to set [29]

$$\bar{\mathcal{X}} = \mathcal{X}_{cl}\ . \tag{35}$$

For the case of a pure dilaton field theory we get for the GEP

$$V^{\mathcal{X}}_{\text{eff}}(\overline{\mathcal{X}}) = V(\overline{\mathcal{X}}) + I^T_1(\Omega^2, T) + \frac{1}{2}(V''(\overline{\mathcal{X}}) - \Omega^2)I^T_0(\Omega^2, T) + \frac{1}{8}V''''(\overline{\mathcal{X}})I^T_0(\Omega^2, T)^2\ , \tag{36}$$

and the variational equation for the determination of Ω reads

$$\frac{1}{2}(V''(\overline{\mathcal{X}}) - \Omega^2) + \frac{1}{4}V''''(\overline{\mathcal{X}})I^T_0(\Omega^2, T) = 0\ . \tag{37}$$

The result is plotted in Fig. 2 for various temperatures and for the two different bag constants. In that figure the singular region at $\overline{\mathcal{X}} = 0$ has been truncated. We see from eq. (36) that the four–point coupling of the $\hat{\mathcal{X}}$–field, which is proportional to $V''''(\overline{\mathcal{X}}) = 24\,C\left[\ln\left(\mathcal{X}/\overline{\mathcal{X}}\right) + 11/6\right]$, develops a singularity at $\overline{\mathcal{X}} = 0$. The expansion of the tree level potential breaks down at the origin. According to the interpretation of the model, the phase $\overline{\mathcal{X}} = 0$ would correspond to a vanishing gluon condensate, and because of eq. (3) also to a vanishing chiral condensate. This means that all of the non–perturbative physics has gone and we are in the pure perturbative region. An interpretation of this phase as the deconfinement phase as done in [4] is therefore natural. But this would mean that the effective degrees of freedom of our model, mesons and glueballs, have been dissolved. Therefore a breakdown of the model at $\overline{\mathcal{X}} = 0$ is only to be expected.

However, we can still use Fig. 2 to extract the temperature scale, where thermal excitations become important, even if we can't explicitly follow the phase transition to $\overline{\mathcal{X}} = 0$. This temperature scale is apparently determined by the chosen bag constant. For $\mathcal{B}^{1/4} = 140$ MeV we begin to see a strong shift of the minimum at temperatures of about $T = 250$ MeV, and above $T = 300$ MeV the minimum at $\overline{\mathcal{X}} \neq 0$ is lost, while for the large bag constant $\mathcal{B}^{1/4} = 240$ MeV the GEP doesn't show any visible shift up to temperatures of order $T \cong 350$ MeV, and only above $T \cong 450$ MeV the minimum at $\overline{\mathcal{X}} \neq 0$ disappears.

We now proceed to the GEP of the full Lagrangian (4), where we choose a SU(N) flavour symmetric mass matrix $\mathcal{M}_q$. The result for the GEP is

$$\begin{aligned} V^{\text{SU(N)}}_{\text{eff}}(\overline{\mathcal{X}}) \;=\; & V^{\Phi}(\overline{\mathcal{X}}) + I^T_1(\Omega^2, T) + \frac{1}{2}[(V^{\Phi})''(\overline{\mathcal{X}}) - \Omega^2]I^T_0(\Omega^2, T) + \frac{1}{8}V''''(\overline{\mathcal{X}})I^T_0(\Omega^2, T)^2 \\ & +(N^2-1)I^T_1(M^2_\pi(\overline{\mathcal{X}}), T) + \frac{(N^2-1)m^2_\pi}{4Nf_\pi}\left(\frac{\mathcal{X}_0}{\overline{\mathcal{X}}}\right)I^T_0(M^2_\pi(\overline{\mathcal{X}}), T)^2 \\ & + \frac{(N^2-1)m^2_\pi}{2\overline{\mathcal{X}}\mathcal{X}_0}I^T_0(M^2_\pi(\overline{\mathcal{X}}), T)I^T_0(\Omega^2, T)\ , \end{aligned} \tag{38}$$

with the condition for Ω

$$\frac{1}{2}[(V^\Phi)''(\overline{\mathcal{X}})-\Omega^2]+\frac{1}{4}V''''(\overline{\mathcal{X}})I_0^T(\Omega^2,\,T)+\frac{(N^2-1)m_\pi^2}{2\overline{\mathcal{X}}\mathcal{X}_0}I_0^T(M_\pi^2(\overline{\mathcal{X}}),\,T)I_0^T(\Omega^2,\,T)=0\,. \quad (39)$$

The result is plotted for $N=2$ in Fig. 3. Qualitatively the GEP behaves similary to the pure dilaton theory, Fig. 2. The dominant contribution to the shape of the GEP (38) as a function of $\overline{\mathcal{X}}$ comes from the one loop term $I_1^T[\Omega^2(\overline{\mathcal{X}}),\,T]$ of the glueball field. (This does not imply the same for the absolute values of the different terms in (38), because the overall normalisation of the curve in Fig.'s 2, 3 was chosen by hand to facilate comparison at different temperatures.) This means that the glueball dynamics itself dominates the position of the minimum of the GEP. The Goldstone bosons and their coupling to the dilaton field play only a minor role.

The leading singularity at the origin, however, is influenced strongly by the four–point coupling of the mesons and the coupling between glueballs and mesons which diverge as $1/\overline{\mathcal{X}}$ as $\overline{\mathcal{X}}\to 0$. Therefore the two last terms in (38) dominate the singular behaviour and change the sign of the pole (now positive) relative to the pure glueball case. These explains the steep rise of the EP curves near the origin at high temperatures.

As a final example we investigate the case of broken flavour $\mathrm{SU(3)_b}$. The result for the GEP is

$$\begin{aligned}
V_{\rm eff}^{\rm SU(3)_f}(\overline{\mathcal{X}}) \;=\; & V^\Phi(\overline{\mathcal{X}})+I_1^T(\Omega^2,\,T)+\frac{1}{2}[(V^\Phi)''(\overline{\mathcal{X}})-\Omega^2]I_0^T(\Omega^2,\,T)+\;\frac{1}{8}V''''(\overline{\mathcal{X}})I_0^T(\Omega^2,\,T)^2\\
& 3I_1^T(M_\pi^2(\overline{\mathcal{X}}),\,T)+4I_1^T(M_K^2(\overline{\mathcal{X}}),\,T)+I_1^T(M_\eta^2(\overline{\mathcal{X}}),\,T)\\
& +\frac{1}{24f_\pi}\left(\frac{\mathcal{X}_0}{\overline{\mathcal{X}}}\right)\Big[9m_\pi^2I_0^T(M_\pi^2(\overline{\mathcal{X}}),\,T)^2-(m_\eta^2+4cm_s)I_0^T(M_\eta^2(\overline{\mathcal{X}}),\,T)^2\\
& \qquad -6m_\pi^2I_0^T(M_\pi^2(\overline{\mathcal{X}}),\,T)I_0^T(M_\eta^2(\overline{\mathcal{X}}),\,T)\\
& \qquad +16m_K^2I_0^T(M_K^2(\overline{\mathcal{X}}),\,T)I_0^T(M_\eta^2(\overline{\mathcal{X}}),\,T)\Big]\\
& +\frac{1}{2\overline{\mathcal{X}}\mathcal{X}_0}I_0^T(\Omega^2,\,T)\Big[3m_\pi^2I_0^T(M_\pi^2(\overline{\mathcal{X}}),\,T)+4m_K^2I_0^T(M_K^2(\overline{\mathcal{X}}),\,T)\\
& \qquad +m_\eta^2I_0^T(M_\eta^2(\overline{\mathcal{X}}),\,T)\Big]\,,
\end{aligned} \quad (40)$$

with

$$\begin{aligned}
&\frac{1}{2}[(V^\Phi)''(\overline{\mathcal{X}})-\Omega^2]+\;\frac{1}{4}V''''(\overline{\mathcal{X}})I_0^T(\Omega^2,\,T)\\
&+\frac{1}{2\overline{\mathcal{X}}\mathcal{X}_0}\Big[3m_\pi^2I_0^T(M_\pi^2(\overline{\mathcal{X}}),\,T)+4m_K^2I_0^T(M_K^2(\overline{\mathcal{X}}),\,T)+m_\eta^2I_0^T(M_\eta^2(\overline{\mathcal{X}}),\,T)\Big]=0\,.
\end{aligned} \quad (41)$$

Here we only have a result for the large value of the bag constant, shown in Fig. 4. The reason is that for the potential $V^\Phi(\mathcal{X})$ (8) the negative curvature due to the $g_\mathcal{X}\mathcal{X}^3$–term is so strong that by varying C and $\mathcal{X}_p$ the bag constant can never be reduced below the value $\mathcal{B}^{1/4}=220$ MeV. The general behaviour of the curves in Fig. 4 is similar to the

former cases. The onset of the thermal shift of the minimum occurs at somewhat lower temperatures; this can be seen better below, when we consider the gluon condensate.

3. CONDENSATES

3.1. The Gluon Condensate

The minimum of the GEP determines the vacuum expectation value $\langle \mathcal{X} \rangle$. This minimum can only be determined numerically. We then use the identification (2) and set

$$\langle \frac{\beta(g)}{2g} G^{a}_{\mu\nu} G^{a\,\mu\nu} \rangle = -C \langle \mathcal{X}^4 \rangle = -C \langle \mathcal{X} \rangle^4 \ , \tag{42}$$

where we use the approximation $\langle O^n \rangle \approx \langle O \rangle^n$, because the EP allows only the determination of $\langle \mathcal{X} \rangle$. The result is shown in Fig. 5 for different parameter sets.

The crucial point is that the gluon condensate is stable up to temperatures of order 200 MeV for both bag constants. We already mentioned that only the higher bag constant is realistic, where the decrease of the gluon condensate sets in even later. However, we must emphasize that our results should be taken only on a very qualitative level above $T = 150$–200 MeV. The reason is that our starting point, the non–linear sigma model, represents only the lowest order of chiral perturbation theory, and the neglecting of higher order gradient terms limits its quantitative applicability to temperatures below the pion mass. Thus the relevant change in the gluon condensate happens only at temperatures where the chiral sector of the model has already broken down. However, the critical temperature for the melting of the gluon condensate is dominated by the glueball dynamics, as can be seen from a comparison in Fig. 5 of the pure glueball scenario with the model including all eight pseudoscalar mesons. For this reason we believe that our statements about the behaviour of the gluon condensate at temperatures above the limit of validity of chiral perturbation theory are at least qualitatively correct, unless the chiral phase transition implies also deconfinement for the glueballs (for which our effective model does not provide any mechanism).

We see in Fig. 5 that the value for T_c is dominated by the value of the bag constant. A larger bag constant results in a higher T_c. At the high–temperature end of the drop in the gluon condensate the curves seem to level off; thus the gluon condensate doesn't completely vanish at T_c, but approaches zero only in the limit $T \to \infty$. The surviving condensate just above the steep drop seems to be largest for the case of $\mathrm{SU}(3)_\mathrm{b}$ with a large strange quark mass. This ties in with the observation made in the introduction that the gluon condensate is strongly correlated to the scale anomaly. The anomaly, a quantum effect, is not expected to vanish at some fixed temperature, in contrast to spontaneously broken symmetries [14, 30]. What we apparently see here is the fading of the condensate relative to a new scale brought in by the temperature.

To be more specific, let us for simplicity look at the pure glueball case. The scale of the anomaly is given by the gluon condensate at $T = 0$ or by the bag constant, which are related in our pure glueball model by

$$\mathcal{B} = -\,\frac{1}{4} \langle \frac{\beta(g)}{2g} G^{a}_{\mu\nu} G^{a\,\mu\nu} \rangle_{T=0} \ . \tag{43}$$

We now read off the critical temperatures for the two investigated bag constants, obtaining $T_c = 290$ MeV for $\mathcal{B}^{1/4} = 140$ MeV and $T_c = 470$ MeV for $\mathcal{B}^{1/4} = 240$ MeV. We thus have

$$T_c \approx 2 \times \mathcal{B}^{1/4} = \sqrt{2}\, \langle \frac{\beta(g)}{2g} G^{\,a}_{\mu\nu} G^{\,a\,\mu\nu} \rangle^{1/4}_{T=0} \,. \tag{44}$$

The result $T_c = \mathcal{O}(\langle G^2 \rangle^{1/4}_{T=0})$ is no surprise, since there is no other relevant scale. (The glueball mass is of minor influence, as will be shown below.) However, it confirms the interpretation, discussed above, that at T_c the temperature becomes the dominating scale, leading to a fading of the gluon condensate. But no complete vanishing is observed, meaning the anomaly stays on at high temperatures. Therefore the non–perturbative physics incorporated in the gluon condensate doesn't vanishes at T_c but stays on to very high temperatures. A continuation of non–perturbative physics beyond the T_c of the Wilson loop is also seen in other sectors, e.g. instanton effects [31] or chromo–magnetic correlations [32].

If we look at the influence of the chiral dynamics on the behaviour of the gluon condensate, we see only minor effects. The general behaviour is that as more mesons are incorporated as more T_c is lowered, but the corresponding shift is very small. A larger effect is seen when we include flavour symmetry breaking in the SU(3) case by turning on the strange quark mass and bringing the kaons and η to their physical mass. This effect is at first surprising, because naively the heavier mesons shouldn't be excited as easily as in the case of exact SU(3) symmetry with (nearly) massless quarks. The reason for the observed effect is the scaling behaviour of the meson masses (21), as we will now discuss.

We determine the gluon condensate via the minimum of the EP. We vary $\overline{\mathcal{X}}$ and determine the energy density. Important are, however, not the absolute contributions, but the variation with $\overline{\mathcal{X}}$, because the global normalisation of the EP is arbitrary. All meson masses scale with $\sqrt{\overline{\mathcal{X}}/\mathcal{X}_0}$. Therefore in the thermal weights we encounter expressions like

$$\mathrm{e}^{-m\sqrt{\overline{\mathcal{X}}/\mathcal{X}_0}/T} \,, \tag{45}$$

which vary more strongly with $\overline{\mathcal{X}}$ the larger the rest mass m is. Thus the location of the minimum is more sensitive to the heavier chiral mesons.

This, of course, raises the question of the influence of all the neglected other heavy hadrons. They surely are important for temperatures of order 200 MeV and higher. Their influence would probably be a shift of T_c to lower values, as indicated above. But the incorporation of these other states into the model is problematic because for these non–Goldstone particles the scaling properties with $\overline{\mathcal{X}}$ are not known. This limits the quantitative power of predictions of our model in the high temperature region.

We summarize by stating that the gluon condensate is very stable up to temperatures above 200 MeV. This is in agreement with other results for the temperature (in–)dependence of the gluon condensate within effective low–energy models [10, 11]. The only exception is the work by Bernard et al. [5], but this is probably due to the very low bag constant taken in their approach.

We now vary the glueball mass at $T = 0$ in order to study the influence of this second empirical value on our results. The gluon condensate as a function of T is shown in Fig. 6 for three values of $m_{\mathcal{X}}$. We see that a lower glueball mass drives T_c to lower values, since lighter glueballs are more easily excited at a given temperature, thus melting the condensate earlier. Quantitatively, however, the effect is small: a variation of the glueball mass by 1 GeV (from 1 GeV to 2 GeV) shifts T_c only by about 50 MeV. Thus the dominant scale for T_c is indeed the bag constant vis. the gluon condensate at $T = 0$.

3.2. The Quark Condensate

We finally investigate the influence of the coupling between dilaton and chiral fields on the chiral condensate. In QCD the chiral condensate is obtained by taking the derivative of the partition function with respect to the current quark masses,

$$\langle q\bar{q}\rangle = \frac{\partial}{\partial m_q} Z^{\mathrm{QCD}} \ . \tag{46}$$

We replace the QCD generating functional by the effective partition function (27)–(29). We can use eq. (26) and rewrite the derivative with respect to m_q into a derivative with respect to m_π. This is equivalent to using the Gell–Mann–Oakes–Renner relation [33]

$$2f_\pi^2 m_\pi^2 = -(m_u + m_d)\langle 0|u\overline{u} + d\overline{d}|0\rangle + \mathcal{O}(m_q^2) \ , \tag{47}$$

which connects the QCD parameters m_q to the empirical parameters f_π, m_π and $\langle q\bar{q}\rangle$ at zero temperature. If we apply $\partial/\partial m_\pi$ to (27)–(29), we need to account for the implicit dependence of $\mathcal{X}_{cl}$ on m_π via eq. (9), which results in the equation

$$(V^\Phi)'(\mathcal{X}_{cl}) = 0 \ . \tag{48}$$

Using this condition we get

$$\frac{\partial \mathcal{X}_{cl}}{\partial m_\pi} = -2\left(\frac{\mathcal{X}_{cl}}{\mathcal{X}_0}\right)^2 \frac{m_\pi f_\pi^2}{M_{\mathcal{X}}^2 \mathcal{X}_0} \approx 6\cdot 10^{-3} \ \text{ for } \mathcal{X}_{cl} = \mathcal{X}_0 \ . \tag{49}$$

Therefore we can neglect this dependence. Similarly the variational equation (39) causes a dependence of Ω on m_π. It is of the same order as (49), and therefore we neglect it, too. We thus get for the chiral condensate

$$\begin{aligned}
\frac{\langle q\bar{q}\rangle^{\mathrm{SU(N)}}(T)}{\langle q\bar{q}\rangle(T=0)} = \left(\frac{\mathcal{X}_{cl}}{\mathcal{X}_0}\right)^3 \Bigg[1 &- \frac{N^2-1}{N f_\pi^2}\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right)^2 \ I_0^T(M_\pi^2(\mathcal{X}_{cl}), T) \\
&- \frac{N^2-1}{2N^2 f_\pi^4}\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right)^4 \ I_0^T(M_\pi^2(\mathcal{X}_{cl}), T)^2 \\
&- \frac{(N^2-1)\mathcal{X}_0^2}{N f_\pi^2 \mathcal{X}_{cl}^4} \ I_0^T(M_\pi^2(\mathcal{X}_{cl}), T) I_0^T(\Omega^2, T) + \frac{3}{\mathcal{X}_{cl}^2} \ I_0^T(\Omega^2(\mathcal{X}_{cl}), T) \\
&+ \frac{(N^2-1)m_\pi^2}{2N^2 f_\pi^4}\left(\frac{\mathcal{X}_0}{\mathcal{X}_{cl}}\right)^3 \ I_0^T(M_\pi^2(\mathcal{X}_{cl}), T) I_{-1}^T(M_\pi^2(\mathcal{X}_{cl}), T) \\
&+ \frac{(N^2-1)m_\pi^2 \mathcal{X}_0}{2N f_\pi^2 \mathcal{X}_{cl}^3} \ I_0^T(\Omega^2, T) I_{-1}^T(M_\pi^2(\mathcal{X}_{cl}), T) \Bigg] \ .
\end{aligned} \tag{50}$$

If we set $(\mathcal{X}_{cl}/\mathcal{X}_0) = 1$ and cancel all terms containing the dilaton mass parameter Ω, we recover the well known result of chiral perturbation theory [34]. The corrections due to the dilaton field are of two kinds. First we have the universal scaling by $(\mathcal{X}_{cl}/\mathcal{X}_0)^3$ as already shown in eq.(3). Second also the $\langle \mathrm{Tr}(U + U^\dagger) \rangle$ factor receives corrections because of the coupling between the dilaton and the chiral field. First there is a scaling of the terms coming from pure meson loops because of the scaling of the meson propagators with negative powers of $(\mathcal{X}_{cl}/\mathcal{X}_0)$ (10). Second there are contributions from glueball loops. These last terms are small as has already been shown in [12].

The quantitativly largest modification comes from the universal scaling, which goes with the third power of $\mathcal{X}_{cl}$. The result is plotted in Fig. 7 for a flavour SU(2), where we set $\mathcal{X}_{cl} = \bar{\mathcal{X}}$ and use the temperature dependence of $\bar{\mathcal{X}}$ and Ω as extracted from the GEP. The solid line corresponds to the result of chiral perturbation theory [34]. The modifications are really small. For the realistic larger bag constant there is nearly no visible shift, because $(\overline{\mathcal{X}}/\mathcal{X}_0) \approx 1$ in the whole temperature region where the chiral condensate is non–zero. Only the smaller bag constant leads to a downward shift of order 20 MeV of the chiral T_c. In general we conclude that the scaling properties of QCD have only a very weak influence on the chiral dynamics.

4. CONCLUSIONS

We have started from an effective Lagrangian which implements the chiral symmetry and scaling aspects of QCD. This allows the investigation of two interesting questions, the thermal behaviour of the gluon condensate and the influence of the scaling properties on the well investigated chiral restoration phenomenon.

We find that the gluon condensate is very stable up to temperatures of 200 MeV, where the chiral sector of the theory reaches its limit of validity. As a result the chiral dynamics is hardly changed at all up to the chiral phase transition. Thermal variations of the gluon condensate and of the chiral condensate occur at two quite different temperature scales. A conclusion from our work is that the gluon condensate does not drive the disappearance of the chiral condensate at high temperature as suggested as one possible scenario in [4].

In our model the melting of the gluon condensate is dictated by the glueball dynamics itself. The influence of the chiral mesons is minor, but other hadron states neglected in our approach may further change the temperature scale of the gluon condensate. This thermal scale is fixed by the value of the condensate at zero temperature or, equivalently, the bag constant. In a pure glueball theory we find $T_c^{\langle GG \rangle} \approx \sqrt{2} \langle GG \rangle_{T=0}^{1/4}$.

It was suggested in [4] that the expectation value $\langle \mathcal{X} \rangle$ of the dilaton field, i.e. the gluon condensate, could be used as an order parameter for gluon deconfinement. Our results contradict this interpretation. We see a relatively stable gluon condensate, well beyond the point of evaporation of the quark condensate which, according to [4], should be interpreted as quark deconfinement. Since we see no reason why one kind of colored particle should be deconfined earlier than another one, we suggest that the color deconfinement phase transition occurs before the gluon condensate vanishes. This implies that nonperturbative phenomena persist well into the deconfined phase.

This is also seen in lattice QCD results, where one is able to extract via some extra-

polation the finite temperature behaviour of the gluon condensate [35]. There one sees at the critical temperature $T_c^{\rm WL}$ of the Wilson loop no evidence for a rapid decrease of the gluon condensate. The authors of [35] suggest that at $T_c^{\rm WL}$ half of the zero temperature condensate survives. All of these analysis suggest that the gluon condensate is not suited as an order parameter for deconfinement.

This conclusion is also supported by the fact that the gluon condensate is strongly connected to the QCD scale anomaly. Quantum–field theoretical anomalies are not expected to vanish at some finite temperature or density. Our results for the gluon condensate seem to support this expectation, since in our calculation the condensate, after a steep decrease at some T_c, levels off at a finite value, suggesting complete disappearance only in the limit $T \to \infty$.

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Appendix

Here we present a list of integrals used in the text. We use the notation of [19]. The zero temperature integrals are defined by

$$I_N^{T=0}(M^2) = \int \frac{\mathrm{d}^3 p}{2E(\vec{p})(2\pi)^3} [E(\vec{p})]^{2N} \ . \tag{51}$$

These and their finite temperature analogues satisfy the following recursion relation:

$$\frac{\mathrm{d}I_N(\Omega^2)}{\mathrm{d}\Omega^2} = \frac{2N-1}{2}\, I_{N-1}(\Omega^2) \ . \tag{52}$$

In this paper we have used the following integrals:

$$I_{-1}(M^2,T) = I_{-1}^{T=0}(M^2) + I_{-1}^{T}(M^2,T) \tag{53}$$

$$I_{-1}^{T=0}(M^2) = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{1}{2E^3(\vec{p})} \tag{54}$$

$$\begin{aligned} I_{-1}^{T}(M^2,T) &= \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{1}{E^2(\vec{p})\,(e^{\beta E(\vec{p})}-1)} \left[\frac{1}{E(\vec{p})} + \frac{\beta e^{\beta E(\vec{p})}}{e^{\beta E(\vec{p})}-1}\right] \\ &= \frac{1}{4\pi^2} \int_{\beta M}^{\infty} \mathrm{d}x \frac{\sqrt{x^2-\beta^2 M^2}}{x^2(e^x-1)^2}\left[(1+x)e^x - 1\right] \end{aligned} \tag{55}$$

$$I_0(M^2,T) = I_0^{T=0}(M^2) + I_0^{T}(M^2,T) \tag{56}$$

$$I_0^{T=0}(M^2) = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{1}{2E(\vec{p})} \tag{57}$$

$$
\begin{aligned}
I_0^T(M^2, T) &= \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{E(\vec{p})}\,\frac{1}{e^{\beta E(\vec{p})}-1} = \frac{T^2}{2\pi^2}\int\limits_{\beta M}^{\infty}\mathrm{d}x\frac{\sqrt{x^2-\beta^2M^2}}{e^x-1} &(58)\\
I_0^T(M^2=0, T) &= \frac{T^2}{12} &(59)\\
I_1(M^2, T) &= I_1^{T=0}(M^2) + I_1^T(M^2, T) &(60)\\
I_1^{T=0}(M^2) &= \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{E(\vec{p})}{2} &(61)\\
I_1^T(M^2, T) &= \frac{1}{\beta}\int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\,\ln\left(1-e^{-\beta E(\vec{p})}\right)\\
&= \frac{T^4}{2\pi^2}\int\limits_{\beta M}^{\infty}\mathrm{d}x\,x\,\sqrt{x^2-\beta^2M^2}\ln\left(1-e^x\right) &(62)\\
I_1^T(M^2=0, T) &= -\frac{\pi^2\,T^4}{90} &(63)
\end{aligned}
$$

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Figure Caption

Figure 1.: Feynman diagrams for the calculation of (24) in first order in the expansion of $\exp\{\mathrm{i}\int \mathrm{d}^4x\mathcal{L}^{\Omega}_{\mathrm{int}}\}$. The solid line corresponds to the glueball field $\hat{\mathcal{X}}$, the long dashed line to Φ^a and the short dashed line to $\partial_\mu\Phi^a$.

Figure 2.: The Gaussian effective potential for the case of only dilaton fields for various temperatures T. The zero temperature glueball mass is $m_{\mathcal{X}} = 1.6$ GeV and $\mathcal{B}^{1/4} = 140$ MeV in a) and $\mathcal{B}^{1/4} = 240$ MeV in b), respectively. The singular region at $\overline{\mathcal{X}} = 0$ has been truncated.

Figure 3.: The Gaussian effective potential for the case of flavour SU(2) in the meson sector for various temperatures T. The zero temperature glueball mass is $m_{\mathcal{X}} = 1.6$ GeV and $\mathcal{B}^{1/4} = 140$ MeV in a) and $\mathcal{B}^{1/4} = 240$ MeV in b), respectively. The singular region at $\overline{\mathcal{X}} = 0$ has been truncated.

Figure 4.: The Gaussian effective potential for the case of broken flavour SU(3) ($m_u = m_d < m_s$) in the meson sector, which represents the physical meson masses, for various temperatures T. The zero temperature glueball mass is $m_{\mathcal{X}} = 1.6$ GeV and $\mathcal{B}^{1/4} = 240$ MeV. The singular region at $\overline{\mathcal{X}} = 0$ has been truncated.

Figure 5.: The gluon condensate as a function of temperature, normalized to its zero temperature value. The solid line corresponds to the scenario of glueball fields only, the long dashed line to flavour SU(2), the long–short dashed line to flavour SU(3), and the short dashed line to broken flavour SU(3) (see Fig. 4). We show results for two bag constants and $m_{\mathcal{X}} = 1.6$ GeV.

Figure 6.: The gluon condensate as a function of temperature, normalized to its zero temperature value. For the meson sector we use an SU(2) symmetric spectrum, and we set $\mathcal{B}^{1/4} = 140$ MeV. The solid line corresponds to $m_{\mathcal{X}} = 2.0$ GeV, the short dashed line to $m_{\mathcal{X}} = 1.6$ GeV and the long dashed line to $m_{\mathcal{X}} = 1.0$ GeV.

Figure 7.: The chiral condensate as a function of temperature, normalized to its zero temperature value. For the meson sector we use an SU(2) symmetric spectrum. The solid line corresponds to the result of chiral perturbation theory [34]. The dashed lines corresponds to the corrections due to the dilaton field. The short dashed line is for $\mathcal{B}^{1/4} = 240$ MeV and the long dashed line for $\mathcal{B}^{1/4} = 140$ MeV.

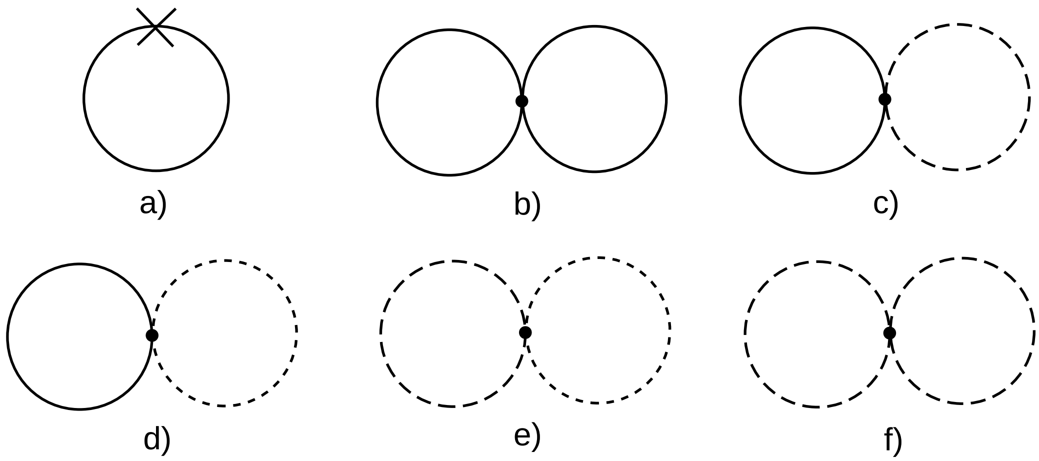
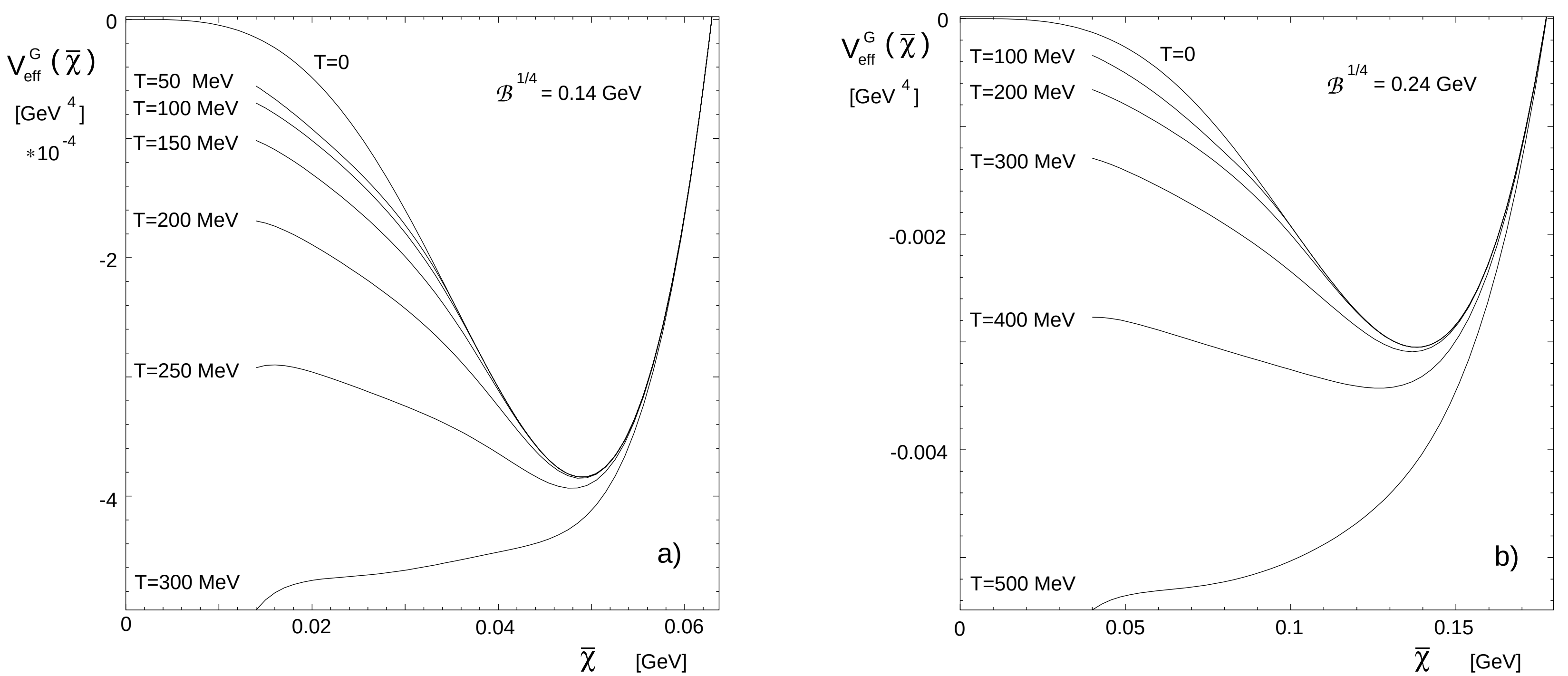


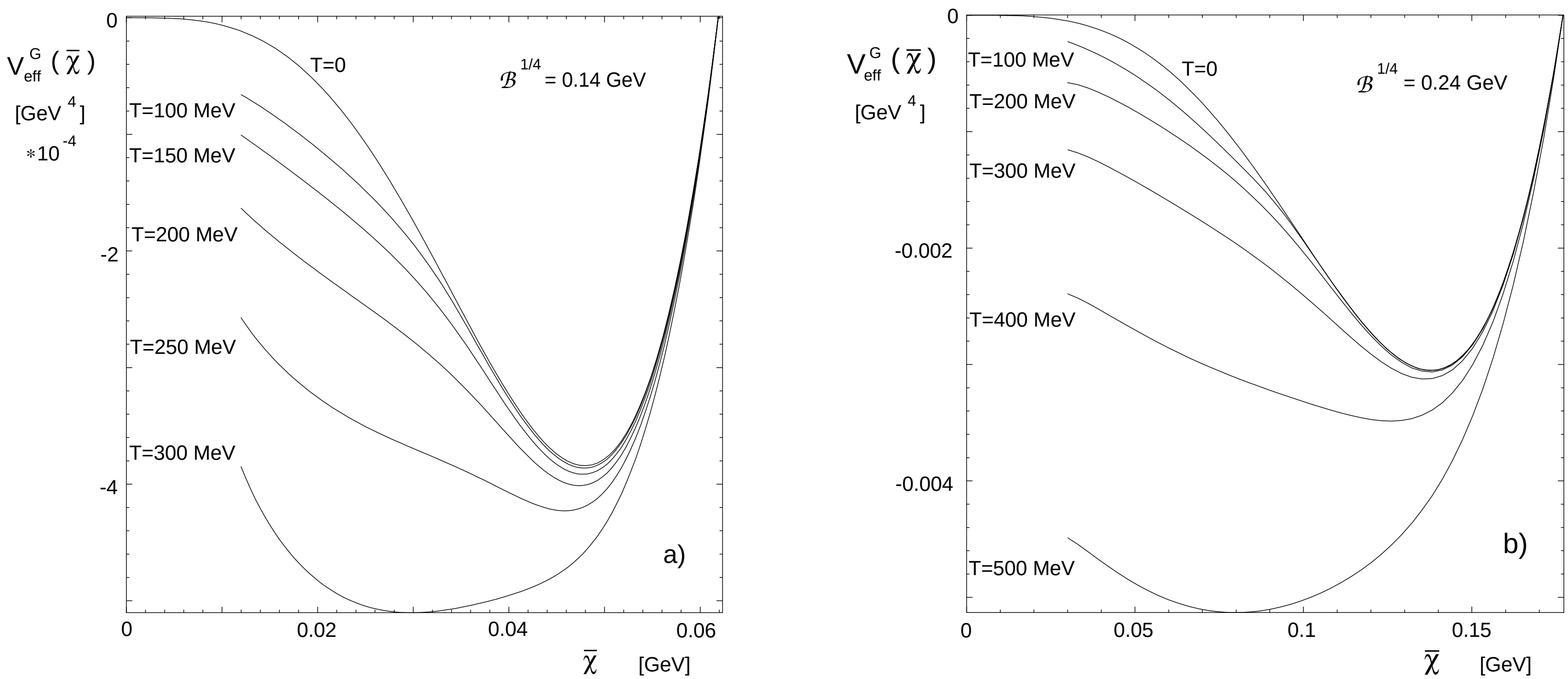
Fig. 1

Fig. 2

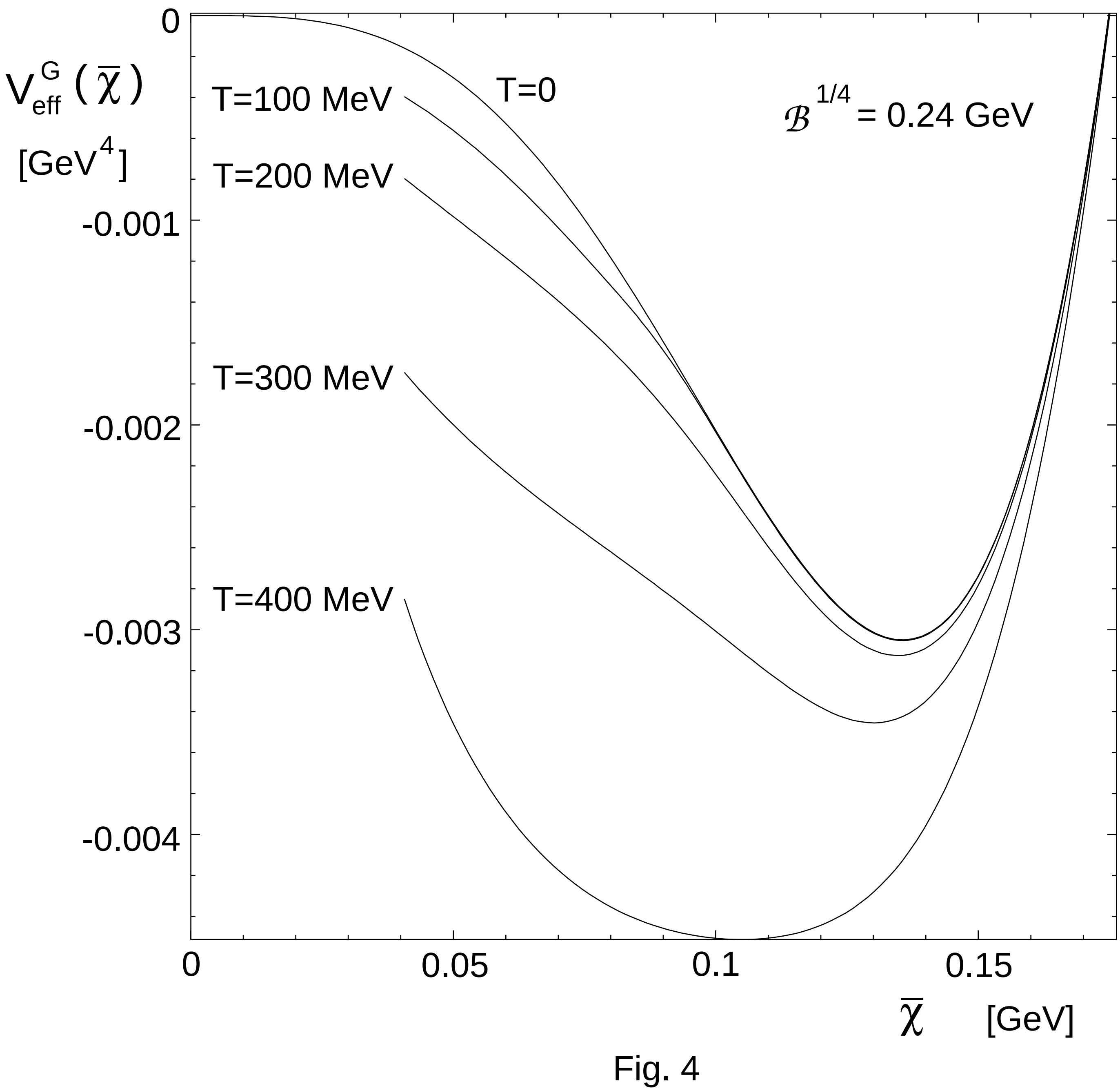


0
$V_{eff}^{G}(\bar{\chi})$
[GeV ^{4}]
$*10^{-4}$
T=0
T=50 MeV
T=100 MeV
T=150 MeV
T=200 MeV
T=250 MeV
T=300 MeV
$\mathcal{B}^{1/4}$ = 0.14 GeV
-2
-4
a)
0
0.02
0.04
0.06
$\bar{\chi}$ [GeV]
0
$V_{eff}^{G}(\bar{\chi})$
[GeV ^{4}]
T=100 MeV
T=0
T=200 MeV
T=300 MeV
T=400 MeV
T=500 MeV
$\mathcal{B}^{1/4}$ = 0.24 GeV
-0.002
-0.004
b)
0
0.05
0.1
0.15
$\bar{\chi}$ [GeV]

Fig. 3

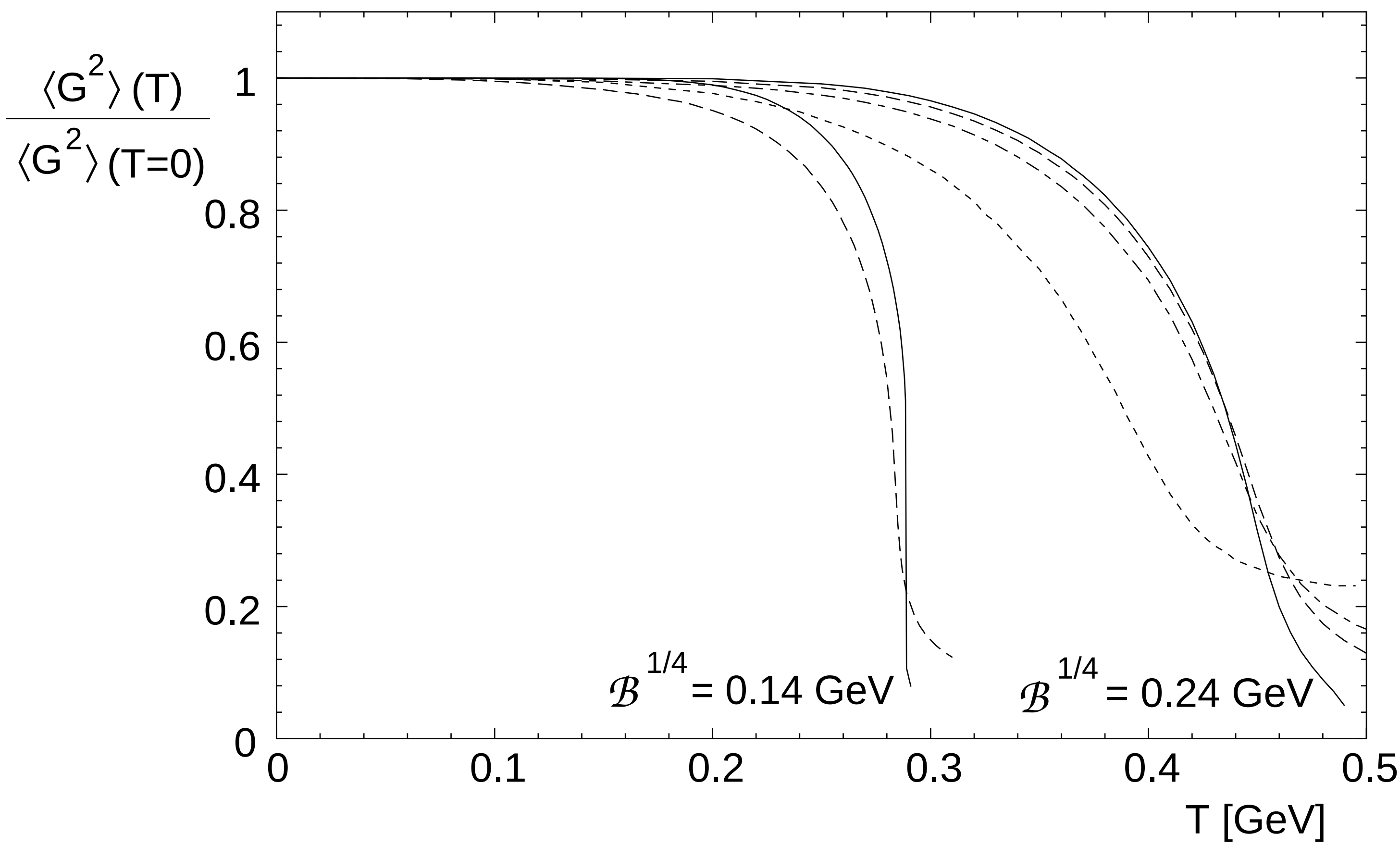


V_eff^G (χ̄)
[GeV 4]
*10 -4
T=0
B 1/4 = 0.14 GeV
T=100 MeV
T=150 MeV
T=200 MeV
T=250 MeV
T=300 MeV
0
-2
-4
0
0.02
0.04
0.06
χ̄ [GeV]
a)
V_eff^G (χ̄)
[GeV 4]
T=0
B 1/4 = 0.24 GeV
T=100 MeV
T=200 MeV
T=300 MeV
T=400 MeV
T=500 MeV
0
-0.002
-0.004
0
0.05
0.1
0.15
χ̄ [GeV]
b)



$V_{eff}^{G}(\bar{\chi})$
[GeV^4]
0
-0.001
-0.002
-0.003
-0.004
T=0
T=100 MeV
T=200 MeV
T=300 MeV
T=400 MeV
$\mathcal{B}^{1/4}$ = 0.24 GeV
0
0.05
0.1
0.15
$\bar{\chi}$ [GeV]

Fig. 4



$\frac{\langle G^2 \rangle (T)}{\langle G^2 \rangle (T=0)}$
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
T [GeV]
$\mathcal{B}^{1/4}$ = 0.14 GeV
$\mathcal{B}^{1/4}$ = 0.24 GeV

Fig. 5

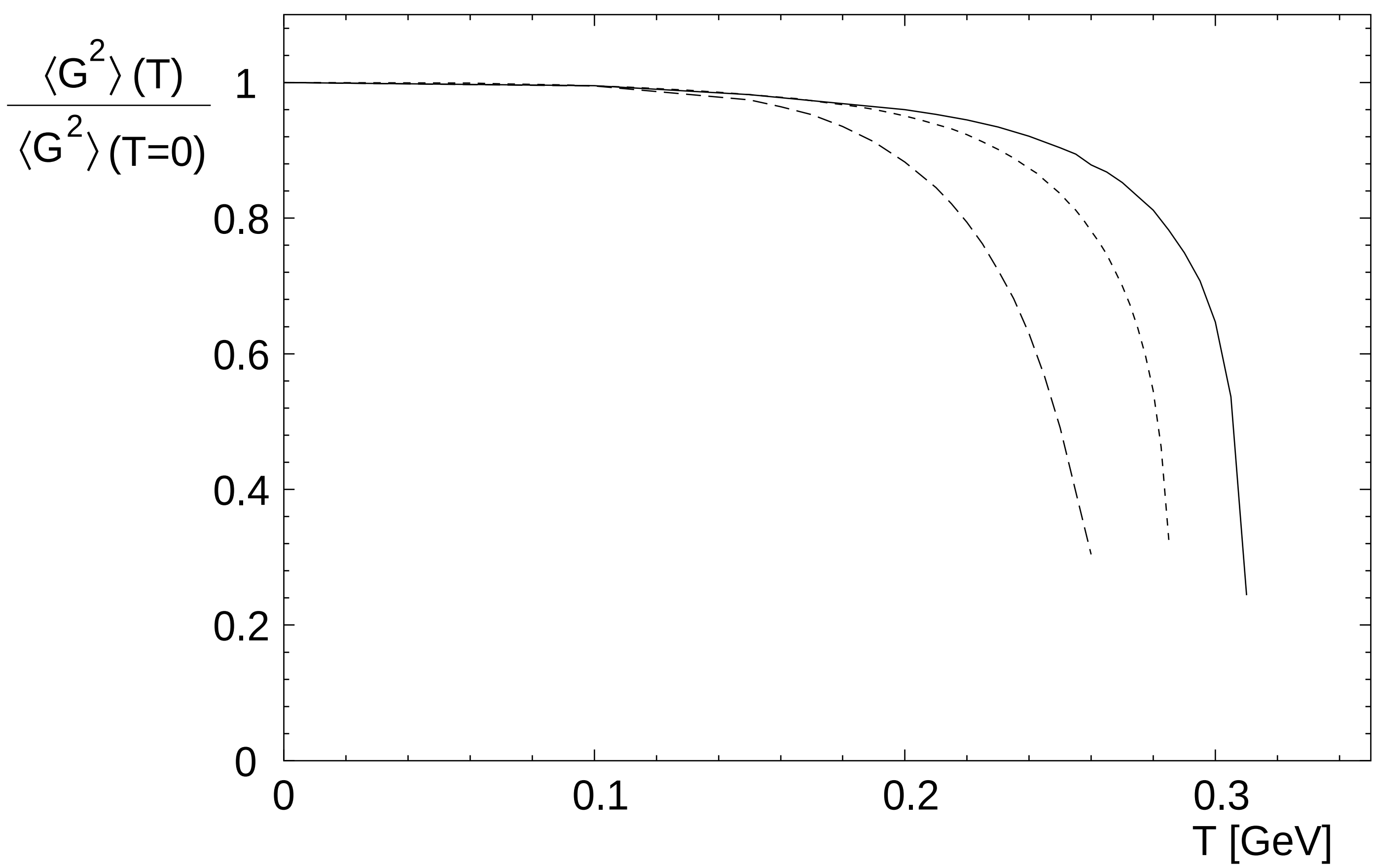

$\frac{\langle G^2 \rangle (T)}{\langle G^2 \rangle (T=0)}$
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
T [GeV]

Fig. 6

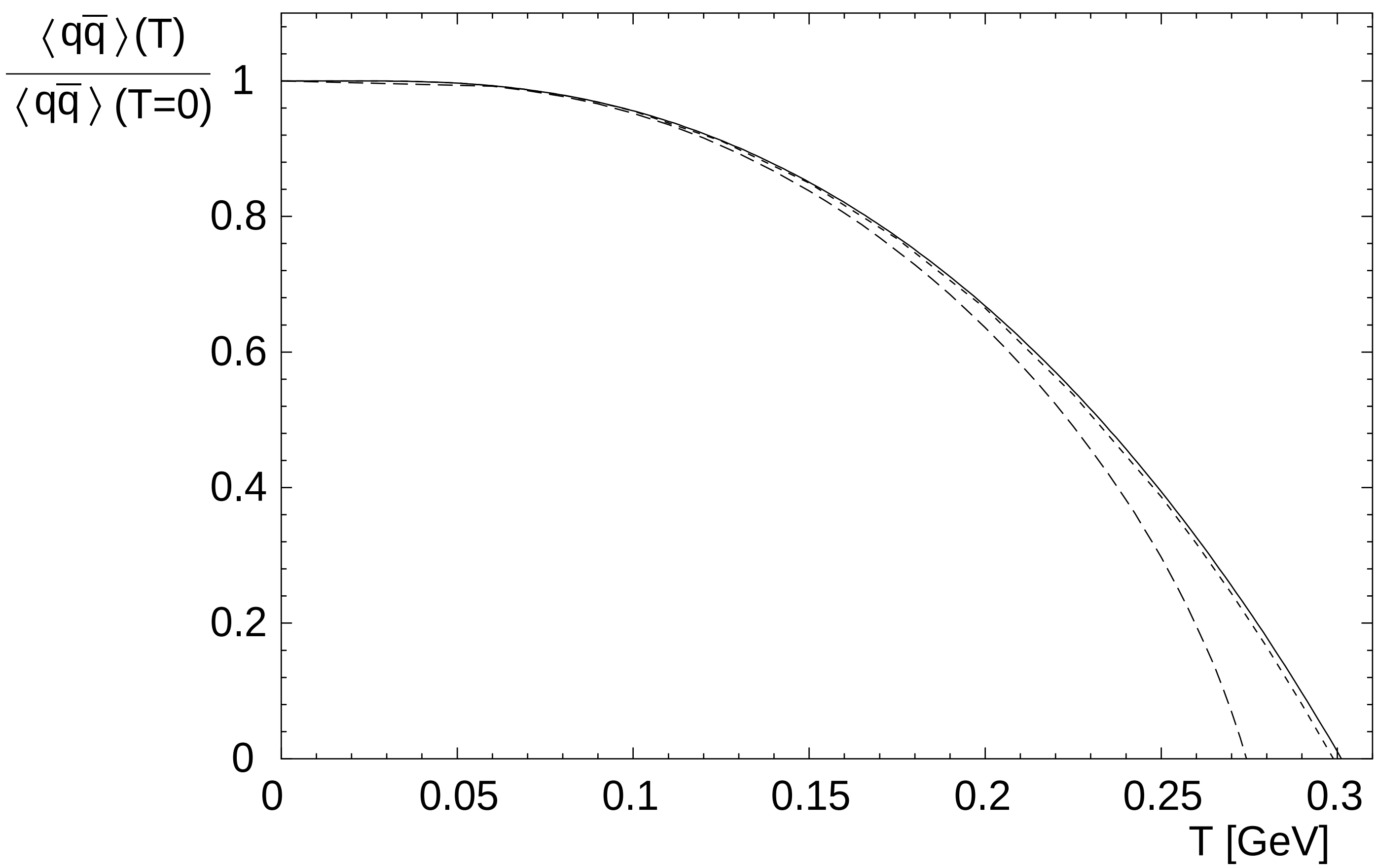

$\frac{\langle q\bar{q} \rangle (T)}{\langle q\bar{q} \rangle (T=0)}$
1
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
0.25
0.3
T [GeV]

Fig. 7